

# Review of "Ole Thomassen Hjortland: "Harmony and the Context of Deducability"

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Hjortland's article formulates a challenge — the "substructural challenge" — for "logical inferentialism". Logical inferentialism is a bundle of doctrines which agree in the core claim that the meaning of logical constants is (or can be) fixed by inference rules which govern their use in proofs. Theories incorporating this claim have been proposed by Dummett (cf. MR0439563 as well as [2]), Martin-Löf (cf. MR1387767, MR0921834), Prawitz (cf. MR0546430, MR0921837, MR2228441), and Schroeder-Heister (cf. MR2926608, MR2933786, MR2228442, MR0676850) among others.

After a brief introduction, Hjortland describes the historical background of inferentialism starting with Prior's [3] famous note, which puts forward an objection against this doctrine, and ending with a brief account of the various proposals towards a regulative principle which overcomes Prior's objection by delimiting the class of proper meaning explanations *via* inference rules. Already Gentzen, in his seminal PhD-thesis (MR1545497 and MR1545507) published in 1934/35, hints at such a principle in a brief remark. There he explains that in the application of an elimination rule to a complex formula $A \circ B$, this formula may only be used as that which it means according to the introduction rule for the connective $\circ$ ("[...] darf die betreffende Formel [...] nur 'als das benutzt werden, was sie auf Grund der Einführung dieses Zeichens bedeutet'"; p. 189 of MR1545497). Thus there must be a certain harmony between introduction and elimination rules. In section 4 of his article, Hjortland presents and explains the principle of "general elimination harmony" (GE-harmony) due to Read [4]. Pior's example of a set of meaning defining rules for a new binary connective *tonk* which, if added to a consistent system, renders that system inconsistent is ruled out by that principle.

The central part of Hjortland's article (sections 5 and 6 and the conclusion) develops an argument which is parallel to that put forward by Prior. It is based on the fact that rules for a connective adhering to the principle of GE-harmony may imply what would be called a structural rule in a sequent system. So-called multiplicative conjunction $\otimes$ provides an example for this. Multiplicative conjunction is the connective characterized (in a sequent system) by Ketonen's (cf. MR0017228) reformulation of Gentzen's original rules.

$$\frac{A, B, \Gamma \Rightarrow C}{A \otimes B, \Gamma \Rightarrow C} \qquad \frac{\Gamma_1, \Rightarrow A \quad \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow A \otimes B}$$

As is readily seen, contraction is derivable from these two rules (and dilution).

Now it is well known that the derivation of Curry's antinomy (cf. MR0007366) requires contraction (in a sequent system or some principle such as $[A \to [A \to B]] \to [A \to B]$ in a Hilbert-type system). Hjortland's argument may then be described as follows. There are formal systems which are (a) consistent, (b) incorporate unrestricted comprehension, but whose underlying logic is (c) substructural since they lack the rule of contraction. According to Curry's result (in MR0007366) extending such a system by multiplicative conjunction will render that system inconsistent (by Curry's antinomy) although natural deduction

rules for $\otimes$ can be set up which conform to the principle of GE-harmony. The addition of $\otimes$ changes what Hjortland, following Belnap [1], calls "the context of deducibility", namely the set of basic assumptions concerning the consequence relation. It is a case of "structural nonconservativeness" (sec. 6 of Hjortland's article). That harmony principles, such as GE-harmony, do not exclude such cases of nonservativeness for the extension of substructural systems is exactly the "substructural challenge" for logical inferentialism.

# References


[1] Belnap, Nuel D: "Tonk, plonk and plink". *Analysis* 22. 130–134. — Reprinted in [5, 132–137].

[2] Dummett, Michael (1991): *The Logical Basis of Metaphysics*. London: Duckworth.

[3] Prior, Arthur (1961): "The runabout inference-ticket". *Analysis* 21. 38–39. — Reprinted in [5, 129–131].

[4] Read, Stephen: "General-elimination harmony and the meaning of the logical constants". *Journal of Philosophical Logic* 39. 123–154.

[5] Strawson, Peter [ed.]: *Philosophical Logic*. Oxford: Oxford University Press.


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